Investment Driven Economic Catch-Up: Revisiting the Growth-Distribution Nexus

Research Proposal of Wending's Third Paper

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1 Introduction

- Two main features of East Asian growth miracle
  - ▶ technology catch-up: allies of the US,
  - ▶ high investment rate.
- Key factors of the high investment rate
  1. financial suppression: buffer stocks due to an extremely incomplete financial market and tight borrowing constraints (we don't focus on this),
  2. very low social welfare: strong precautionary saving motives of households,
  3. state investment: infrastructure, heavy industry,
  4. young population structure: more savers, fewer consumers.
- Characteristics of investment-driven growth:
  - ▶ high saving rate, low consumption rate,
  - ▶ severe inequality,
  - ▶ sustained current account surplus.
- Impact of economic catch-up on the developed country:
  - ▶ lower interest rate due to huge capital supply from the developing country,
  - ▶ higher growth rate due to investment in the developing country (?)
  - ▶ increasing inequality due to the lower labor income share in output (?) and high social welfare.

2 Model

We build a two-country model to characterize the economic catch-up process. There is one developing country denoted by the subscript $\mathfrak{p}$ and one developed country denoted by the subscript $\mathfrak{r}$. Both countries produce the same traded goods. The capital is mobile across countries, but the labor is immobile.

Time is discrete. For any time t, there are $a = 1$ to $a_{\max}$ age groups in each country. Each agent lives and works for $a_{\max}$ periods. For each age group at t in the country $m \in \{\mathfrak{p}, \mathfrak{r}\}$, the agents whose ages are a at t are indexed by $i = 1, \cdots, I_{m,t,a}$. We denote the population of the country m at time t as $I_{m,t} \equiv \sum_{a=1}^{a_{\max}} I_{m,t,a}$.

2.1 Production

Both countries use Cobb-Douglas production functions to produce goods. The production functions are:

$$Y_{m,t} = A_{m,t} K_{m,t}^{\alpha} L_{m,t}^{1-\alpha}, \tag{1}$$

where the country index $m \in \{\mathfrak{p}, \mathfrak{r}\}$. Notice that the only difference between the two countries' production functions is the TFP level $A_{m,t}$.

2.1.1 Technology in Developed Country

The TFP process in the developed country is given by:

$$A_{\mathfrak{r},t} = A_{\mathfrak{r},0}(1+g)^t U_{\mathfrak{r},t}, \tag{2}$$

where $A_{\mathfrak{r},0}$ is the initial TFP level of the developed country, g is the stable TFP growth rate at the technological frontier, $U_{\mathfrak{r},t}$ is the productivity shock following a first-order Markov process capturing the business cycle.

2.1.2 Technology in Developing Country

Following Fernández-Villaverde et al. (2023), we use a catch-up technology process to characterize the TFP process of a developing country:

$$A_{\mathfrak{p},t} = \theta A_{r,0} \gamma_t (1+g)^t U_{p,t}, \tag{3}$$

$$\log(\gamma_t) = \rho \log(\gamma_{t-1}), \tag{4}$$

where we denote θ as the asymptotic share of the technological frontier the developing country can achieve. We use $\gamma_t \in (0,1)$ to measure the distance between the developing country's technology and the technology frontier, and the initial technology gap γ_0 is given, and $0 < \rho < 1$ in (4) captures the curvature of catch-up path. Notice that $A_{\mathfrak{p},t} \xrightarrow{d} \theta A_{\mathfrak{r},0}(1+g)^t U_{p,t}$ as $t \to \infty$. A first-order Markov process $U_{\mathfrak{p},t}$ is used to characterize the business cycle in the developing country.

2.1.3 Labor and Capital Demand

We denote δ as the depreciation rate of capital in both sectors.

The competitive wage for labor input and net interest rate in the two countries $m \in \{f, p\}$ are given by:

$$w_{m,t} = (1-\alpha)A_{m,t}\left(\frac{K_{m,t}}{L_{m,t}}\right)^{\alpha} \tag{5}$$

$$r_{m,t} = \alpha A_{m,t}\left(\frac{L_{m,t}}{K_{m,t}}\right)^{1-\alpha} - \delta. \tag{6}$$

2.2 Demography

The birth rate $\theta_{m,t}$ in country $m \in \{\mathfrak{p}, \mathfrak{r}\}$ is a deterministic function of per capita GDP, $\frac{Y_{m,t}}{L_{m,t}}$. We treat per capita GDP as a proxy for all the factors that affect the birth rate, including urbanization, medical and education levels, etc. In both countries, an agent randomly matches with another agent at $a = 1$, i.e., when they enter the labor market and form a household. The number of kids $n_{m,t} \in \{0,1,2,3,4,5\}$ in each household follows a categorical distribution $\text{Cat}(n_{m,t} \mid p_{m,t,0}, \ldots, p_{m,t,5})$, where $\sum_{n_{m,t} \geq 1} p_{m,t,n_{m,t}} \cdot n_{m,t} \equiv \theta_{m,t}$ such that the law of large numbers holds.

2.3 Government

2.3.1 Government in the Developed Country

The government in the developed country collects capital and income tax to provide social insurance. The government's budget constraint is:

$$G_{\mathfrak{r},t} + C_{\mathfrak{r},t} = T_{\mathfrak{r},t}, \tag{7}$$

where $C_{\mathfrak{r},t}$ is the government consumption at time t, which is used to balance the government budget.

2.3.2 Government in the Developing Country

The government in the developing country also collects capital and income tax, but it uses the tax revenue to invest in the domestic economy as well as provide social insurance. Moreover, we let the total capital stock in the developing country $K_{\mathfrak{p},t} := K_{\mathfrak{p},s,t} + K_{\mathfrak{p},h,t}$, where $K_{\mathfrak{p},s,t}$ is the amount of capital held by the state, and $K_{\mathfrak{p},h,t}$ is the amount of capital held by the households. The government uses tax income to invest as well as to provide social insurance. The government's budget constraint is:

$$G_{\mathfrak{r},t} + (1+\psi)K_{\mathfrak{r},t+1} = T_{\mathfrak{r},t} + (1+r_{\mathfrak{r},t})K_t^s, \tag{8}$$

where T_t is the tax income at time t, $G_{\mathfrak{r},t}$ is the spending on social insurance at t, ψ is a discount rate reflecting the low efficiency of the government's investment.

2.4 Households

We denote the aggregate states in the economy as $S_t = (K_{\mathfrak{r},t}, K_{\mathfrak{p},t}, L_{\mathfrak{r},t}, L_{\mathfrak{p},t}, I_{\mathfrak{p},t}, I_{\mathfrak{r},t}, U_{\mathfrak{r},t}, U_{\mathfrak{p},t})$. The agent living in country m with age a at time t solves the DP problem:

$$\begin{aligned}
&\omega_{m,t,a}\Big(x_{m,t,a}, l_{m,t,a}; S_t\Big) = \max_{\substack{c_{m,t,a},\\ k_{\mathfrak{r},t+1,a+1},\\ k_{\mathfrak{p},t+1,a+1}}} \Big\{u(c_{m,t,a}) + \beta\mathbb{E}_t\omega_{m,t+1,a+1}\Big(x_{m,t+1,a+1}, l_{m,t+1,a+1}; S_{t+1}\Big)\Big\},\\
&\text{s.t. } x_{m,t+1,a} = c_{m,t+1,a} + k_{\mathfrak{r},t+1,a+1} + k_{\mathfrak{p},t+1,a+1},\\
&x_{m,t+1,a+1} = \max\big\{\underline{x},\ (1-\tau_{m,k})\big[(1+r_{\mathfrak{r},t+1})k_{\mathfrak{r},t+1,a+1} + (1+r_{\mathfrak{p},t+1})k_{\mathfrak{p},t+1,a+1}\big] + (1-\tau_{m,l})w_{m,t+1}l_{m,t+1,a+1}\big\},\\
&l_{m,t,a} = q(a) + \mu_{t,a},\\
&K_{\mathfrak{r},t+1} = \mathcal{P}_{\mathfrak{r},k}(S_t),\\
&K_{\mathfrak{p},t+1} = \mathcal{P}_{\mathfrak{p},k}(S_t),\\
&L_{\mathfrak{r},t+1} = \mathcal{P}_{\mathfrak{r},l}(S_t),\\
&L_{\mathfrak{p},t+1} = \mathcal{P}_{\mathfrak{p},l}(S_t),\\
&\omega_{m,t+a_{\max}-1,a_{\max}}\Big(x, l; S\Big) = \max_{0\le c\le x}\big\{d\big[\xi u(c) + (1-\xi)u(x^i_{a_{\max}} - c_{a_{\max}} + e)\big] + (1-d)u(c)\big\},
\end{aligned} \tag{9}$$

where $x_{m,t,a}$ is the wealth level, $l_{m,t,a}$ is the labor supply at t; $c_{m,t,a}$ is the consumption level at t, $k_{\mathfrak{r},t+1,a+1}$, $k_{\mathfrak{p},t+1,a+1}$ are the holdings of the two country capital at the start of $t+1$; $\underline{x}$ is the social insurance level; $\tau_{m,k}$ is the capital income tax rate for country m citizens, $\tau_{m,l}$ is the labor income tax rate for country m citizens; the permanent labor supply $q(a)$ is a quadratic function of age, and $\mu_{a,t}$ is the AR(1) process of individual labor supply shock. The prediction function $\mathcal{P}(S_t)$ is a function of the aggregate states in the economy, which can accurately forecast next period aggregate capital stock and labor supply. Notice that we can use the law of large numbers to predict $L_{m,t+1}$, and linear regression model to predict $K_{m,t+1}$. The exogenous dummy variable $d = 1$ if the agent has some kids and $d = 0$ otherwise. The coefficient $1 - \xi$ is the weight of the bequest utility, the constant e reflects the luxury good property of bequest (De Nardi et al., 2010).

2.5 Recursive Equilibrium

The equilibrium is characterized by (i) the endogenous labor and capital prices, (ii) the prediction function, and (iii) the set of policy functions for people in different age groups.

1. Firm optimizes profit according to (5) and (6).
2. Agents solve their DP problem (9).
3. Given initial distribution of wealth at $t = 0$, for each period t, total capital supply equals total capital demand.
4. The government's budget (8) holds.
5. The prediction function $\mathcal{P}$ forecasts the next period's aggregate capital stock accurately.

3 Algorithm

We use the algorithm in Krusell and Smith (1998) to solve the model for a given initial distribution of population structure and wealth.

1. Given a guess of $\mathcal{P}$. (maybe an affine function of S_t)
2. Solve agents' DP problem at t.
3. Given policy function, solve the equilibrium prices by market clearing conditions.
4. Simulate the next 1000 periods' wealth distribution and aggregate variables.
5. Check whether $\mathcal{P}$ is consistent with simulated data by looking at R^2, if not, then update $\mathcal{P}$.
6. Repeat steps 2 - 5 until $\mathcal{P}$ converges.

Now we examine these steps in detail. First, we introduce the DP solver assuming the prediction functions are accurate. Then, we discuss how to solve the equilibrium prices given the DP solver. Finally, we show how to find the accurate prediction functions.

3.1 DP Solver

Notice that in the stochastic DP problem (9), there are two choice variables and many exogenous state variables, which are difficult to solve using the conventional backward induction method. We use the *policy tree descendant* (PTD) algorithm (Iskhakov and Liu, 2024) to solve (9).

To use the PTD algorithm, we first consider the deterministic counterpart of the original stochastic DP problem. We fix the basic uncertainties $\{U_{\mathfrak{r},j}, U_{\mathfrak{p},j}, \mu_{j,a}\}_{j=t}^{t+a_{max}-1}$ in (9), as a result, the factor prices series $\{r_{\mathfrak{r},j}, r_{\mathfrak{p},j}, w_{m,j}\}_{j=t}^{t+a_{max}-1}$ are deterministic. Since we must have $r_{\mathfrak{r},t} \geq r_{\mathfrak{p},t}$ or vice versa, the agent buys at most one country's capital in each period. Thus, the deterministic dynamic portfolio choice problem becomes a deterministic consumption-saving problem with one-asset and social insurance discussed in Iskhakov and Liu (2024):

$$\begin{aligned}
&v_{m,t,a}(x_{m,t,a}) = \max_{c_{m,t,a}} \Big\{ u(c_{m,t,a}) + \beta v_{m,t+1,a+1}\big(\max\{z_{m,t+1,a+1}, \underline{x}\}\big)\Big\},\\
\text{s.t. }& k_{t,a} = x_{m,t,a} - c_{m,t,a},\\
&z_{m,t+1,a+1} = (1-\tau_{m,k})\big(1+\max\{r_{\mathfrak{r},t+1}, r_{\mathfrak{p},t+1}\}\big)k_{t,a} + (1-\tau_{m,l})w_{m,t+1,a+1}l_{m,t+1,a+1},\\
&0 \leq c_{m,t,a} \leq x_{m,t,a},\\
&v_{m,t+a_{\max}-1,a_{\max}}(x) = \max_{0\leq c\leq x} \big\{ d\big[\xi u(c) + (1-\xi)u(x- \ c+e)\big] + (1-d)u(c)\big\}.
\end{aligned} \tag{10}$$

Based on the deterministic Bellman equation (10), the stochastic Bellman equation (9) can be reformulated as:

$$\begin{aligned}
&\omega_{m,t,a}\Big(x_{m,t,a}, l_{m,t,a}; S_t\Big) = \max_{\substack{c_{m,t,a},\\ k_{\mathfrak{r},t+1,a+1},\\ k_{\mathfrak{p},t+1,a+1}}} \Big\{ u(c_{m,t,a}) + \beta \mathbb{E}_t v_{m,t+1,a+1}\Big(x_{m,t+1,a+1}\Big)\Big\},\\
&\text{s.t. } x_{m,t+1,a} = c_{m,t+1,a} + k_{\mathfrak{r},t+1,a+1} + k_{\mathfrak{p},t+1,a+1},\\
&x_{m,t+1,a+1} = \max\big\{\underline{x},\ (1-\tau_{m,k})\big[(1+r_{\mathfrak{r},t+1})k_{\mathfrak{r},t+1,a+1} + (1+r_{\mathfrak{p},t+1})k_{\mathfrak{p},t+1,a+1}\big] + (1-\tau_{m,l})w_{m,t+1}l_{m,t+1,a+1}\big\},\\
&l_{m,t,a} = q(a) + \mu_{t,a},\\
&K_{\mathfrak{r},t+1} = \mathcal{P}_{\mathfrak{r},k}(S_t),\\
&K_{\mathfrak{p},t+1} = \mathcal{P}_{\mathfrak{p},k}(S_t),\\
&L_{\mathfrak{r},t+1} = \mathcal{P}_{\mathfrak{r},l}(S_t),\\
&L_{\mathfrak{p},t+1} = \mathcal{P}_{\mathfrak{p},l}(S_t),\\
&v_{m,t+a_{\max}-1,a_{\max}}\Big(x\Big) = \max_{0\leq c\leq x} \big\{ d\big[\xi u(c) + (1-\xi)u(x- \ c+e)\big] + (1-d)u(c)\big\}.
\end{aligned} \tag{11}$$

Notice that in (11), the expectation is taken over the basic uncertainties $\{U_{\mathfrak{r},j}, U_{\mathfrak{p},j}, \mu_{j,a}\}_{j=t}^{t+a_{max}-1}$, which can be calculated through Monte Carlo integration. There are two Euler equations based on (11):

$$u'(c_{m,t,a}) = \beta \mathbb{E}_t \Big((1 + r_{\mathfrak{r},t+1}) u'(c_{m,t+1,a+1}) \Big), \tag{12}$$

$$u'(c_{m,t,a}) = \beta \mathbb{E}_t \Big((1 + r_{\mathfrak{p},t+1}) u'(c_{m,t+1,a+1}) \Big). \tag{13}$$

To evaluate $c_{m,t,a}(x_{m,t,a}, l_{m,t,a}; S_t)$, we need to consider four possibilities:

1. $k_{\mathfrak{r},t+1,a+1} > 0$ and $k_{\mathfrak{p},t+1,a+1} > 0$, i.e., we need to solve (12) and (13) simultaneously.

2. $k_{\mathfrak{r},t+1,a+1} > 0$ and $k_{\mathfrak{p},t+1,a+1} = 0$, i.e., we only need to solve (12).

3. $k_{\mathfrak{r},t+1,a+1} = 0$ and $k_{\mathfrak{p},t+1,a+1} > 0$, i.e., we only need to solve (13).

4. $k_{\mathfrak{r},t+1,a+1} = k_{\mathfrak{p},t+1,a+1} = 0$, i.e., the agent plans to reply on social insurance at $t + 1$ and buys no asset.

We can pin down the optimal policy by comparing the values $\omega_{m,t,a}\Big(x_{m,t,a}, l_{m,t,a}; S_t\Big)$ in (11).

3.2 Equilibrium Prices

3.3 Prediction Function

Predicting the aggregate labor supply by the law of large numbers is relatively easy. We only need to specify the initial distribution of labor supply when $a = 1$, then we can use the total population $I_{m,t}$ and aggregate labor supply $L_{m,t}$ to predict $L_{m,t+1}$ for both countries.

Predicting the aggregate capital stocks is difficult.

4 Estimation

References

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